# Have you thought about Sudoku solution？

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**Abstract**

The following essay introduces an increasingly popular puzzle (Sudoku) around the world. Overall, Sudoku benefits us a lot from both logic and health aspects, and it is also a cute puzzle for people, even there are large-scale competitions around the world every year. However, what basic rules and principles are behind this "simple" puzzle, and how to solve the puzzle rapidly with various interesting methods, is discussed in this essay. The methods are classified into two types ---- Intuition techniques and Candidate number techniques. These methods are introduced and illustrated in the essay. What is more, it is interesting that some of these methods can be applied in other types of puzzles, and there are some examples to show you how to achieve this. However, there are still some defects. For example, some words are incorrect, and the application stage has not clearly demonstrated the deep connection between Sudoku and other puzzles. Maybe I would do some alternations as my academic level has progressed.

**Keywords**

Sudoku；Method；Application；Benefits.

## Introduction

Can people who have a weak foundation in Mathematics be good at Sudoku? Are more given numbers more accessible than those fewer ones? Can all Sudoku be solved without guessing? Sudoku is not directly related to math because it is a game based on logic instead of math. Surprisingly, some Sudoku has more given numbers are more complicated than less one and guess sometimes is a crucial stage to solve Sudoku.[i] It is interesting that it cost lots of time for researchers to find what are the least given number for a Sudoku to be unique, the current result shows that 17 given numbers are the least number to make the Sudoku with unique solution [ii] . The 18th-century Swiss mathematician Leonhard Euler initially developed the "Latin Squares" concept, where numbers in a grid appear only once, across and up and down. In the late 1970s, Dell Magazines in the US began publishing what we now call Sudoku puzzles using Euler's concept with a nine by nine square grid. They called it Number Place, and it was developed by an independent puzzle maker, Howard Garnes.[iii] Nowadays, Sudoku has developed into a puzzle game, much like a crossword puzzle. The puzzle itself is nothing more than a grid of little boxes called "cells". The puzzle comes with some of the cells (usually less than half of them) already filled in, like this:

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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 6 | | 5 | 9 | 3 | | | |
| 9 | | 1 | | | | 5 | | |
| | 3 | | 4 | | | | 9 | |
| 1 | | 8 | | 2 | | | | 4 |
| 4 | | | 3 | | 9 | | | 1 |
| 2 | | | | 1 | | 6 | | 9 |
| | 8 | | | | 6 | | 2 | |
| | | 4 | | | | 8 | | 7 |
| | | | 7 | 8 | 5 | | 1 | |

1.1

There are numerous manners to solve the Sudoku. The following essay aims to introduce some techniques involving essential solutions ---- hidden simple, naked simple and advanced solution----locked candidates, subset, and the following essay is going to focus on those techniques. Firstly, the case studies and relative development will be mentioned in my literature review. Secondly, I would integrate some effective manners online and give my own opinion and understanding, which is crucial for us to manage when dealing with those sudokus.

**Literature review**

The word "Sudoku" is another name for "number place". Derived from the Japanese language, "Sudoku" refers to a logic-based placement. A type of puzzle game, in other words. The first Sudoku puzzle was published in 1979 in a US magazine. The puzzle travelled to Japan, where it exploded with popularity. If translated into English phrases, the name "Sudoku" would be "the digits must remain single", though the word Sudoku is an abbreviation of the original Japanese name. The name has gathered various manners of pronunciation in different countries, but the basic standards of the game have not been altered. Retired architect and freelance puzzle constructor Howard Garns created Sudoku while in his late 70s. Mr Garns created puzzles for a living, so he knew the tricks to create a great puzzle. The idea for Sudoku was derived from that of Euler's "Latin Square" puzzle. Howard Garns took the ancient game and altered it by adding a "third dimension". Garns also provided the player with a partially completed grid that required the solver to complete the rest of the blank grids in the Sudoku version.

After arriving in Japan, the name "Sudoku" got applied to the game, which Garns had named "Number Place." Later, the name was copyrighted by a Japanese company, though the puzzle was launched in Britain and then worldwide.

For the first few years, players used paper and pencil to play the game, but in 1989, a computerized version was introduced. More recently, a Rubik's Cube style version of Sudoku has been constructed.

In the year 1997, the popularity of Sudoku in the media had reached overwhelming heights. A Hong Kong judge named Wayne Gould became so engulfed by the game, he developed his computerized version of a Sudoku creator.

Nationwide, newspapers would print the puzzles and sell out quickly. World media acknowledged Sudoku as the "fastest-growing puzzle in the world". By 2005, Sudoku was at the peak of its popularity. The World Sudoku Championship is the largest Sudoku competition globally, sponsored by the World Federation of Puzzles. The first Sudoku World Championship was held in Lucca, Italy, in 2006 and will be held once a year after that, which attracts lots of Sudoku lovers. [iv] Most websites have similar descriptions about the history, so it seems reliable.

Many interesting questions behind Sudoku also attract lots of researchers. For instance, What is the smallest number of clues that can possibly be given such that a sudoku puzzle still has only one solution? However, nobody has found any 16-clue puzzles, and it was conjectured that the answer to the sudoku minimum number of clues problem is 17.

The strategy we used to finally solve this problem is an obvious one — exhaustively search through all possible solution grids, one by one, for a 16-clue puzzle. So we took the point of view of considering each particular completed sudoku grid one at a time, and then looking for puzzles whose solution is that particular grid. We think of these puzzles as being “contained in” that particular grid. Our search turned up no proper 16-clue puzzles, but had one existed, then we would have found it.

A brute force exhaustive search would not have been feasible, but we have developed a novel algorithm that made the exhaustive search possible. Our programme, named checker, improves greatly over the original open-source version of checker from 2006, which would have been far too slow to search all sudoku solution grids. Indeed, the paper [18] estimates that our original version would take over 300,000 years on one computer to finish this project.

It is worth noting that there have been attempts to solve the minimum number of clues problem using mathematics only, i.e., not using a computer. However, nobody has made any serious progress. In fact, while it is very easy to see that a sudoku puzzle with seven clues will always have multiple completions, because the two missing digits can be interchanged in any solution, finding a theoretical reason why eight clues are not enough for a unique solution already seems hard. This is far from the conjectured answer of 17, so a purely mathematical solution of the minimum number of clues problem is a long way off. [v]

As for the checker mentioned above, the work on this project began over six years ago. Back in August 2005, we started writing checker, which to our knowledge was the first computer programme that made it possible to search a sudoku solution grid exhaustively for all n-clue puzzles, where both the grid and the number n were supplied by the user. The last release of this original, open-source version of checker that we posted on the Internet is of November 2006 [2]. In 2009 we started working on a completely new implementation of checker optimized for the case n = 16, which up to now we had never published anywhere. This new version of checker takes only a few seconds to search an average sudoku solution grid for all 16-clue puzzles, whereas the last release from 2006 takes about an hour per grid on average. (Mcguire G, Tugemann B, Civario G. There is no 16-Clue Sudoku: Solving the Sudoku Minimum Number of Clues Problem[J]. 2012). This dissertation was 10 years ago, it should be some new progresses in this field, but the information involved in the paper is worth to understand.

Furthermore, how many different Sudoku exists in total is a solved question, maybe you cannot image how big the number is. By considering all possibilities in each blocks and reduction (Lexicographical reduction, permutation reduction and column reduction), We computed that this particular choice of top three rows can be completed to a full grid in 72 × 97961464 = 7053225408 ways. The same calculations were made for all of the 44 possibilities on our list. This then works out, all at once, how many ways there are to complete 178848 of the 2612736 possibilities for the top rows. We do the same for the other 43, to find exactly how many ways we can complete all of the top three rows to a full grid. This computes the number of grids with B1 in standard form; we need to multiply by 9! = 362880 to get the total number.

In total, there are N0 = 6670903752021072936960 ≈ 6.671 × 1021 valid Sudoku grids. Taking out the factors of 9! and 722 coming from relabelling and the lexicographical reduction of the top row of blocks B2 and B3, and of the left column of blocks B4 and B7, this leaves 3546146300288 = 27 × 27704267971 arrangements, the last factor being prime. Subsequently, Ed Russell verified the result; it has now been verified by several other people as well. [vi] Besides, some researchers tried to find the connection between Sudoku and other things. For example, in the dissertation: Permutation matrices related to Sudoku, Let n be a positive integer, and N = n2. We consider real N × N matrices partitioned into n2 blocks (each being a square matrix of order n) in the following way. Define the set of positions. Skl={(i,j) :(k-1)n<i<=kn,(1-1)n<j<=ln} (1<=k,l<=n) and partition a N×N matrix P accordingly (Dahl G . Permutation matrices related to Sudoku[J]. Linear Algebra and its Applications, 2009, 430(8):2457-2463.). [vii] Actually, there already have had an exactly data and result about the topics. This is an example about how to find the whole possibilities.

The principles and mathematical structure of Sudoku can be found in other fields. The mathematical structure of Sudoku puzzles is akin to hard constraint satisfaction problems lying at the basis of many applications, including protein folding and the ground-state problem of glassy spin systems. Via an exact mapping of Sudoku into a deterministic, continuous-time dynamical system, here we show that the difficulty of Sudoku translates into transient chaotic behavior exhibited by this system. We also show that the escape rate k, an invariant of transient chaos, provides a scalar measure of the puzzle's hardness that correlates well with human difficulty ratings. Accordingly, g 5 2log10 k can be used to define a ''Richter'' - type scale for puzzle hardness, with easy puzzles having 0 , g # 1, medium ones 1 , g # 2, hard with 2 , g # 3 and ultra-hard with g . 3. To our best knowledge, there are no known puzzles with g . 4. [viii] What's more, the dissertation (Image Encryption using the Sudoku Matrix) introduces a new effective and lossless image encryption algorithm using a Sudoku Matrix to scramble and encrypt the image. [ix]

Even, some researchers created other games based on Sudoku. The present invention is concerned with a game board and system for playing multiple Sudoku type problem-solving games. The system comprises a three-dimensional board having a first array of n2 identical cavities arranged in a square matrix and at least one second array totaling at least n2 identical cavities arranged in at least n series of n cavities, at least n identical series of n pegs adapted to fit in said cavities in a plurality of definite angular positions, each peg defining a body having opposite first and second end faces, each peg in a series having its first face bearing one of a series of n different indicia. The body preferably has a polygonal cross-section and the system may further comprise a plurality of rings adapted to slide about the perimeter of said body. A method of playing Sudoku type problem-solving games using the system is further provided. [x] Similarly, a gaming system and method for administering a Sudoku-based game is provided that includes at least one engine for randomly generating a Sudoku puzzle, at least one first module for modifying the generated Sudoku puzzle into a game puzzle, at least one second module for permuting the game puzzle for each player, and at least one system for providing a game puzzle to each player and administering a gaming game from the game puzzle provided to each player. [xi]

The previous works were various and complicated enough. The following essay aims to briefly illustrate several methods to solve Sudoku. I suppose it is not too difficult based on previous researches.

## Intuition techniques

The intuition technique means we solve the Sudoku directly without guessing and aids. The intuition technique is just primary exclusive methods like unique solution, basic exclusion method, block exclusion method, only remainder solution, rectangular exclusion method, unit exclusion method and remainder test method.

### Unique solution

When the number of squares filled with numbers in a particular line reaches 8, the number that can be filled in the remaining squares of the line is only the number that has not appeared yet. Become the unique row solution. When the number of squares filled with numbers in a column reaches 8, the number that can be filled in the remaining squares is only the number that has not appeared yet. Become the unique solution of the column. When the number of squares filled with numbers in a certain nine squares reaches eight, the number that can be filled in the remaining squares is only the one that has not appeared yet. Become the only solution of Jiu Gong Ge. The following gragh3.1.1 is an example: Eight numbers have been added to line a, and only the number 3 has not appeared in line a, so A9=3, which is the only solution of the line.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 8 | 2 | 5 | 9 | 7 | 6 | 4 | |
| B | 3 | 4 | | | 2 | 6 | 9 | | |
| C | 9 | | | | 4 | | | | 2 |
| D | 4 | | | | | 9 | 3 | 2 | 7 |
| E | 8 | 1 | 9 | | | 2 | 4 | 5 | 6 |
| F | 2 | 7 | 3 | 4 | 6 | 5 | 1 | 8 | 9 |
| G | | 9 | 4 | 2 | | | | | |
| H | | 2 | | 9 | | | 7 | | 4 |
| I | 7 | 3 | 8 | 6 | 5 | 4 | 2 | 9 | 1 |

3.1.1

In 3.1.2, eight numbers have been added to the first column, and only the number 5 has not appeared in the first column, so E1=5, which is the unique solution of the column.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 7 | 6 | 2 | 9 | 8 | 5 | 134 | 14 | 34 |
| B | 3 | 9 | 1 | 7 | 246 | 246 | 8 | 246 | 5 |
| C | 4 | 5 | 8 | 3 | 126 | 126 | 7 | 9 | 26 |
| D | 1 | 27 | 9 | 245 | 3 | 246 | 246 | 245 | 246 |
| E | ? | 23 | 345 | 124 | 7 | 8 | 124 | 124 | 246 |
| F | 8 | 27 | 6 | 124 | 124 | 9 | 124 | 3 | 247 |
| G | 9 | 4 | 37 | 268 | 5 | 26 | 236 | 267 | 1 |
| H | 2 | 123 | 35 | 124 | 124 | 7 | 234 | 246 | 9 |
| I | 6 | 8 | 7 | 124 | 9 | 3 | 5 | 24 | 24 |

3.1.2

In diagram3.1.3, eight numbers have been added to the Jiu Gong Ge area where A8 is located, but only the number 9 has not appeared, so A8=9, which is the only solution of Jiu Gong Ge.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 5 | | | | | | 4 | ? | 8 |
| B | | | 6 | | | | 7 | 5 | 2 |
| C | | 9 | 4 | | | | 1 | 3 | 6 |
| D | 9 | 8 | | | | | 6 | | 4 |
| E | 6 | | | | | | 3 | | 9 |
| F | | | 3 | | | | | | |
| G | 2 | | 9 | | | | | | |
| H | | | | | 7 | | | 4 | |
| I | | | | | | 8 | 9 | | |

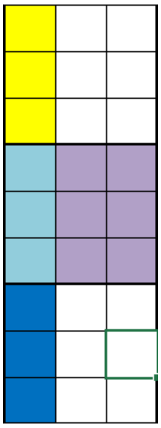
3.1.3

**Basic exclusion method**

The primary exclusion method is to solve problems by using the rule that the numbers from 1 to 9 can only appear once in every row, column and nine squares. The primary exclusion method can be divided into row exclusion, row exclusion and nine squares exclusion. The actual process of finding a solution is: Looking for the nine-square-grid exclusion solution: finding the situation that there is only one number left in a specific nine-square-grid location; That is to say, the filling position of the number in the nine squares is found. Looking for the column exclusion solution: it is found that there is only one left in the position where a particular number can be filled in a column; That is to say, the filling position of the number in the column is found. Looking for row exclusion solution: finding the situation that there is only one number left in a row that can be filled in; That is to say, the filling position of the number in the line is found. Solving problems using the primary exclusion method is to find the only position in which the number 1 ~ 9 can be put in the row, column and nine squares in turn. It is necessary to comprehensively use the methods of row exclusion, column exclusion and nine squares exclusion.

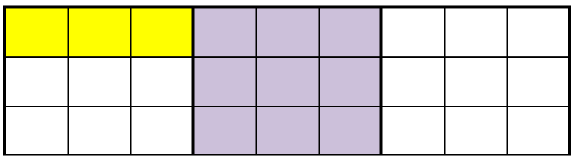
**Block exclusion method**

Block exclusion method is the promotion method of primary exclusion method, and it is one of the most frequently used methods in visual method. The so-called block divides the row into three small connected squares, and the column is also divided into three small connected squares. Jiu Gong Ge is also regarded as composed of three small connected squares, as shown in the following schematic diagram:



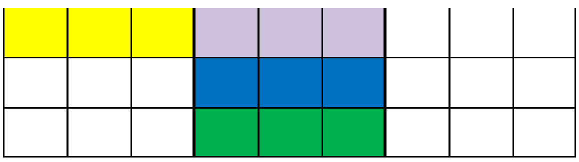
3.3.1

The core idea of the block exclusion method is explained as follows (taking behaviour as an example), and the same is true for the column.



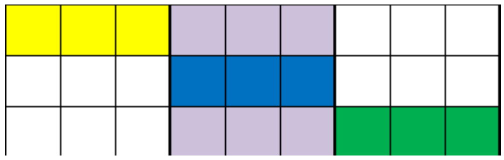
3.3.2

**L**et us say that one of the yellow area blocks (G1~G3) is the number 9.



3.3.3

Then, the blue area of (H4~H6) may contain the number 9. Otherwise, the green area (I4~I6) contains the number 9.

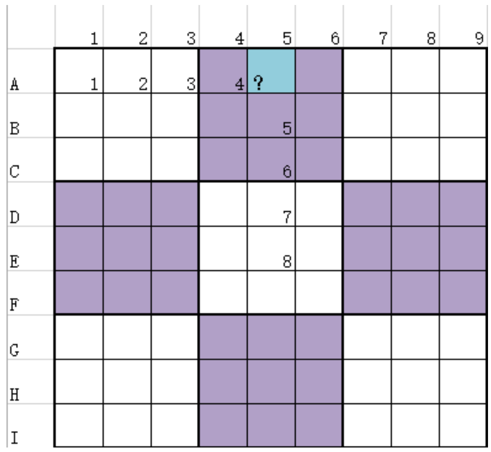


3.3.4

Assume that we have determined that one of the yellow area blocks (G1~G3) is the number 9. (H4~H6). The blue area contains the number 9. Then, the green area (I7~I9) must contain the number 9. If it is determined by other methods that two squares in the (I7~I9) green area cannot be the number 9, then the specific position of the number 9 in the (I7~I9) block can be determined.

**Only remainder solution**

The remainder solution is that the number that can be added to a square has already ruled out 8, so the number of this square can only be added to the number that does not appear.

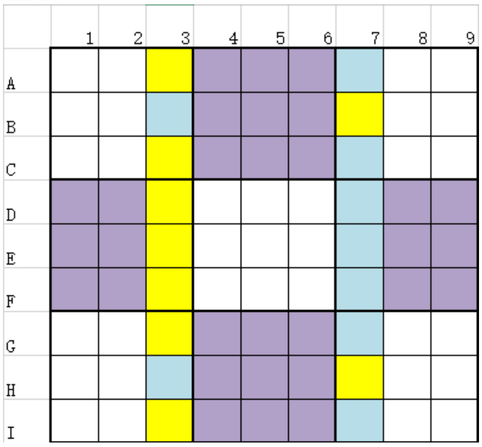



3.4.1

As shown in the above figure, according to the numbers appearing in the row and column, it can be known that only the number 9 can be filled in.

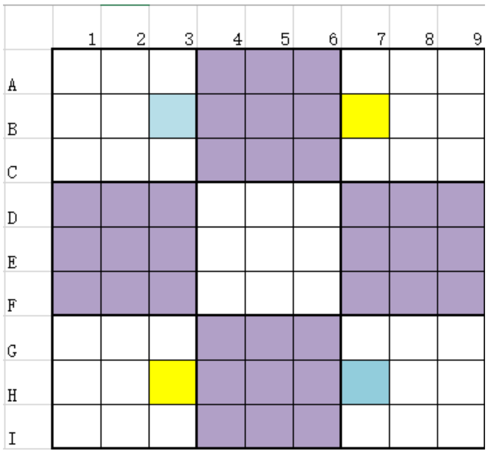
**Rectangular exclusion method**

The rectangular exclusion method is advanced. Although the rectangular exclusion method principle is elementary, it is not easy to observe in practical use. The principle of the rectangular exclusion method is as follows:



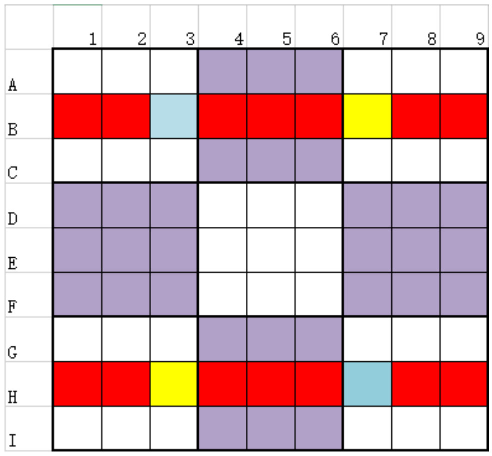

3.5.1

As gragh3.5.1 shown above, if it is in the third column, we determine that the number 9 can only appear in B3 or H3. In column 7, the number 9 can only appear in B7 or H7. B3, H3, B7, H7 form a rectangle, which meets the conditions of the rectangle exclusion method.




3.5.2

From the above, it can be concluded that the number "9" can only appear on (B3, H7) or (B7, H3)

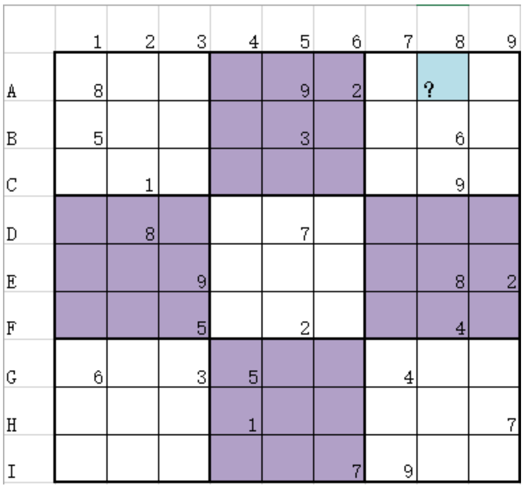



3.5.3

No matter which case appears above, we can infer that the red area of line B and line H can no longer be the number 9.

**Unit exclusion method**

Unit exclusion method is a primary exclusion method, and it is also the easiest to apply and identify.




3.6.1

Can the number of A8 be determined? The answer is yes.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 8 | | | | 9 | 2 | | ? | |
| B | 5 | | | | 3 | | | 6 | |
| C | | 1 | | | | | | 9 | |
| D | | 8 | | | 7 | | | | |
| E | | | 9 | | | | | 8 | 2 |
| F | | | 5 | | 2 | | | 4 | |
| G | 6 | | 3 | 5 | | | 2 | | |
| H | | | | 1 | | | | | 7 |
| I | | | | | | 7 | 9 | | |

3.6.2

From D5=7, D8 is not equal to 7 H9=7, and it is concluded that G8, H8 and I8 are not equal to 7. Obviously A8=7

**Residual test method**

The so-called remainder test method is a problem-solving method in which there are more numbers filled in nine squares in a specific row or column, and when there are two or three remaining squares, the remaining squares are added with values for testing.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | | 3 | 2 | 9 | | | | 7 | |
| B | 1 | 4 | | 7 | 3 | 8 | 9 | | 2 |
| C | 7 | 9 | 8 | 6 | 4 | 2 | | | |
| D | 8 | 6 | 3 | 1 | 2 | 4 | 5 | 9 | 7 |
| E | | 5 | 1 | 3 | | | | | |
| F | | 2 | 7 | 8 | | | | 3 | 1 |
| G | | | | 2 | | 3 | 7 | 4 | |
| H | 2 | | 4 | 5 | | 6 | | | |
| I | 3 | | | 4 | 8 | | | | 6 |

3.7.1

Let us look at lines B and B3 may add 5 or 6. Let us test from 5.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | | 3 | 2 | 9 | | | | 7 | |
| B | 1 | 4 | | 7 | 3 | 8 | 9 | | 2 |
| C | 7 | 9 | 8 | 6 | 4 | 2 | | | |
| D | 8 | 6 | 3 | 1 | 2 | 4 | 5 | 9 | 7 |
| E | | 5 | 1 | 3 | | | | | |
| F | | 2 | 7 | 8 | | | | 3 | 1 |
| G | | | | 2 | | 3 | 7 | 4 | |
| H | 2 | | 4 | 5 | | 6 | | | |
| I | 3 | | | 4 | 8 | | | | 6 |

3.7.2

We add 5 to B3 for testing and get the left picture, but no wrong inference is obtained so that B3=5 may be the correct judgment. If we can judge that B3 is not 6, we can be sure that B3=5.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 6 | 3 | 2 | 9 | | | | 7 | |
| B | 1 | 4 | 5 | 7 | 3 | 8 | 9 | 6 | 2 |
| C | 7 | 9 | 8 | 6 | 4 | 2 | | | |
| D | 8 | 6 | 3 | 1 | 2 | 4 | 5 | 9 | 7 |
| E | | 5 | 1 | 3 | | | | | |
| F | | 2 | 7 | 8 | | | | 3 | 1 |
| G | 5 | | 6 | 2 | | 3 | 7 | 4 | |
| H | 2 | | 4 | 5 | | 6 | | | |
| I | 3 | | 9 | 4 | 8 | | | | 6 |

3.7.3

So we also need to test with B3=6, Add 6 to B3 and push B8=5.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | | 3 | 2 | 9 | | | | 7 | |
| B | 1 | 4 | 6 | 7 | 3 | 8 | 9 | 5 | 2 |
| C | 7 | 9 | 8 | 6 | 4 | 2 | | | |
| D | 8 | 6 | 3 | 1 | 2 | 4 | 5 | 9 | 7 |
| E | | 5 | 1 | 3 | | | | | |
| F | | 2 | 7 | 8 | | | | 3 | 1 |
| G | | | | 2 | | 3 | 7 | 4 | |
| H | 2 | | 4 | 5 | | 6 | | | |
| I | 3 | | | 4 | 8 | | | | 6 |

3.7.4

Look at lines c, C7, C8 and C9 must contain the number 5. It is proved that B3=6 is wrong. And thus B3=5

## Candidate number techniques

Candidate number refers to all the numbers that can be filled in a grid. The candidate number skill is to judge the correct number by associating the candidate numbers in the grid. It is a more complex method than the intuitive method, all of which are also called advanced solution, which contains unique candidate number method, unique implicit candidate number method, block deletion method, number pair deletion method, implicit number pair deletion method, triple chain number deletion method, implicit triple chain number deletion method, rectangular vertex deletion method, triple chain column deletion method and key number deletion method

### Unique candidate number method

When there is only one number of candidates in a square lattice, this number is the only one in the square lattice. This candidate number is solving the problem with the candidate number method, that is, the process of gradually eliminating inappropriate candidates.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | | | 1 | | | 5 | | 7 | |
| B | | 2 | 5 | | 6 | | | | |
| C | | | 9 | | 1 | | | | |
| D | | 6 | 12345 6789 | | | | 7 | | |
| E | | 3 | 8 | | | | 2 | | |
| F | 2 | 5 | | 9 | | | | | 1 |
| G | | | | 1 | | | | 3 | |
| H | 4 | | | | | | | 6 | 2 |
| I | 7 | | | 5 | | 2 | | | |

4.1.1

We can rule out the possibility that D3 is 12356789. After the safe deletion of the candidate number, the candidate number of D3 becomes the only candidate number of "4".

### Implicit unique candidate number method

When a certain number appears only once in the candidate number of each column, then this number is the only candidate number of this column. The value of this square can be determined as the number. At this time, according to the rules of the Sudoku game, each column should contain the number 1 ~ 9, and the candidates of other squares do not contain this number, so this number cannot appear in other squares, so it can only appear in this square. If the only candidate number appears in rows and nine squares, the processing method is the same. This is a prepared candidate list. Pay attention to B5, B9 and D1

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 28 | 458 | 9 | 6 | 25 | 12458 | 127 | 3 | 57 |
| B | 238 | 3568 | 1 | 7 | 235 | 258 | 26 | 4 | 569 |
| C | 7 | 3456 | 23456 | 12345 | 9 | 1245 | 126 | 8 | 56 |
| D | 39 | 7 | 346 | 239 | 8 | 26 | 5 | 16 | 346 |
| E | 1 | 35689 | 3568 | 359 | 4 | 567 | 3678 | 2 | 3678 |
| F | 38 | 2 | 34568 | 35 | 1 | 567 | 9 | 67 | 34678 |
| G | 5 | 138 | 2378 | 1248 | 267 | 9 | 34678 | 67 | 34678 |
| H | 6 | 189 | 78 | 1458 | 57 | 3 | 478 | 579 | 2 |
| I | 4 | 369 | 2378 | 258 | 2567 | 25678 | 3678 | 5679 | 1 |

4.2.1

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 28 | 458 | 9 | 6 | 25 | 12458 | 127 | 3 | 57 |
| B | 238 | 3568 | 1 | 7 | 235 | 258 | 26 | 4 | 569 |
| C | 7 | 3456 | 23456 | 12345 | 9 | 1245 | 126 | 8 | 56 |
| D | 39 | 7 | 346 | 239 | 8 | 26 | 5 | 16 | 346 |
| E | 1 | 35689 | 3568 | 359 | 4 | 567 | 3678 | 2 | 3678 |
| F | 38 | 2 | 34568 | 35 | 1 | 567 | 9 | 67 | 34678 |
| G | 5 | 138 | 2378 | 1248 | 267 | 9 | 34678 | 67 | 34678 |
| H | 6 | 189 | 78 | 1458 | 57 | 3 | 478 | 579 | 2 |
| I | 4 | 369 | 2378 | 258 | 2567 | 25678 | 3678 | 5679 | 1 |

4.2.2

It can be seen that in the first column, the number 9 only appears in D1. In the fifth column, the number 3 appears only in B2. In the Jiu Gong Ge, where B9 is located, the number 9 only appears in B9. Therefore, "9" is the invisible only candidate in the first column. "3" is the unique invisible candidate in the fifth column. "9" is the invisible only candidate for A7 Jiu Gong Ge. Therefore, it is determined that D1 = 3, B5 = 3 and B9 = 9.

**Three-chain number deletion method**

The method of finding out that there are no more than three different numbers among three candidates in a particular column, a specific row or a certain nine squares, and then deleting these three numbers from the candidates in other squares is called the triple chain number deletion method. The principle of the triple chain number deletion method is shown below

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| G | | | | | | | | | |
| H | | 12 | | | 23 | | 13 | | |
| I | | | | | | | | | |

4.3.1

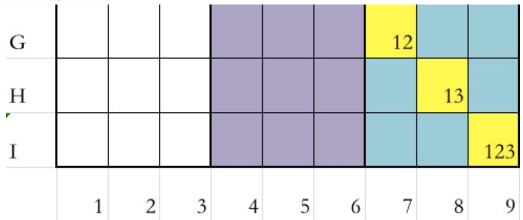
Inline H, the candidate numbers (12), (23) and (13) of H2, H5 and H7 constitute triple chain numbers, so the three numbers of 123 can only appear inline H, and these three candidate numbers can be deleted in other houses of the bank. This is the case where the triple chain number occurs in the line.

Inline H, the candidate numbers (12), (23) and (13) of H2, H5 and H7 constitute triple chain numbers, so the three numbers of 123 can only appear inline H, and these three candidate numbers can be deleted in other houses of the bank. This is the case where the triple chain number occurs in the line.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| G | | | | | | | 12 | | |
| H | | | | | | | | 23 | |
| I | | | | | | | | | 13 |

4.3.2

In the Jiu Gong Ge where G7 is located, the candidate numbers (12), (23) and (13) of G7, H8 and I9 constitute a triple chain number, so the three numbers of 123 can only appear in G7, H8 and I9 in this Jiu Gong Ge, and these three candidate numbers can be deleted in other squares of this Jiu Gong Ge. This is the case where the triple chain number occurs in Jiu Gong Ge.

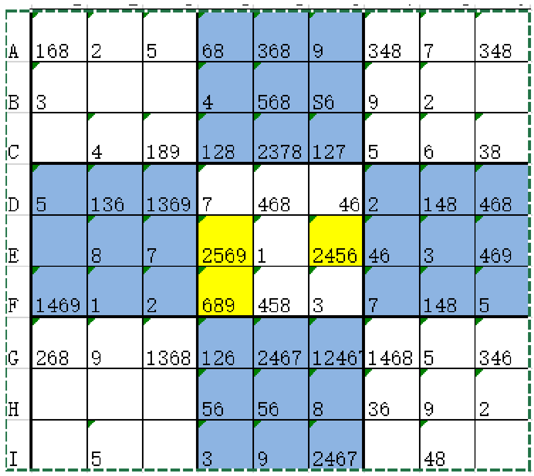



4.3.3

Triple chain number is an extension of number pairs. We extend the above triple chain number to get the unique triple chain number on the right. As long as it is guaranteed to be in three squares and contains three candidates, it all meets our requirements, such as (123,123,123) and (12, 12,123). It can be found that as long as the number of candidates contained in n squares is just n, then the processing is the same as the number of triple chains, thus forming the number of four chains, such as (12, 23, 34, 14), (123, 123, 14, 1234), etc. Therefore, it can be extended to five chains or more, but those are not often used.

**Implicit triple chain number deletion method**

Implicit triple chain numbers are developed from implicit number pairs. Three numbers appear in the same grid in a sure line, but none of the other grids in this line contains these three numbers. We call this number pair an invisible triple chain number. Then all the other numbers in the candidates of these three squares can be excluded.
4. Implicit triple chain number deletion method
Implicit triple chain numbers are developed from implicit number pairs. Three numbers appear in the same grid in a specific line, but none of the other grids in this line contains these three numbers. We call this number pair an invisible triple chain number. Then all the other numbers in the candidates of these three squares can be excluded. The treatment method is the same when the number of invisible triple chains appears in the column and nine squares. We further expand that N numbers appear in the same square in a row (column, nine squares), and other squares in this bank do not contain these N numbers. We call this number pair an invisible N chain number. Then other numbers in the candidate numbers of the N squares can be excluded

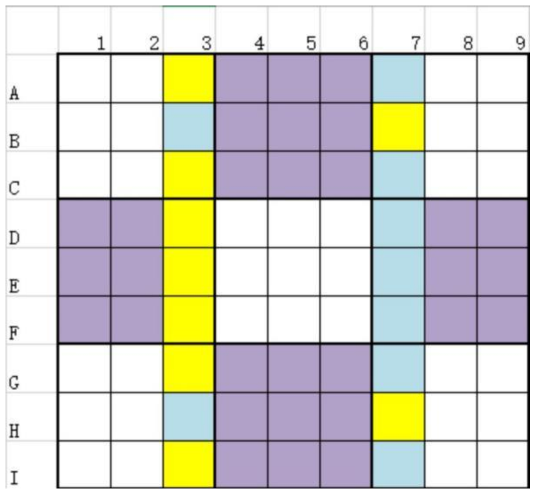



4.4.1

In the middle nine squares, the candidate numbers "2", "5" and "9" only appear in E4, E6 and F4, forming an invisible triple chain number, so in E4, E6 and F4, other candidate numbers can be excluded, and F4=9.

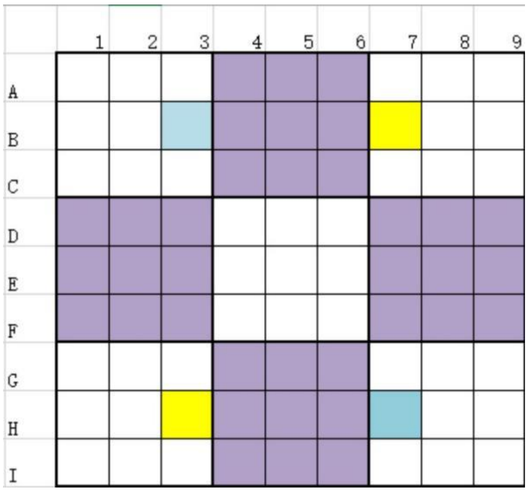
**Rectangular vertex deletion method**

The rectangular vertex deletion method is the same as the rectangular exclusion method. Rectangular vertex deletion is not easy to find in recognition, so it is better to use other methods first.



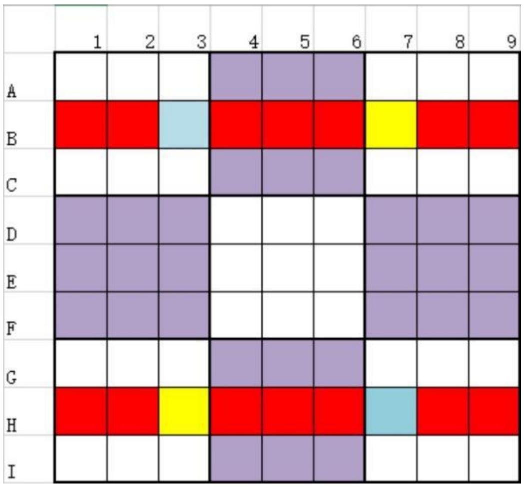

4.5.1

As shown above, if in the third column, the candidate number "9" can only appear in B3 or H3. In the 7th column, the candidate number "9" can only appear in B7 or H7. B3, H3, B7, H7 form a rectangle, which meets the conditions of the rectangle vertex deletion method.




4.5.2

From the above, it can be concluded that the number "9" can only appear on (B3, H7) or (B7, H3)

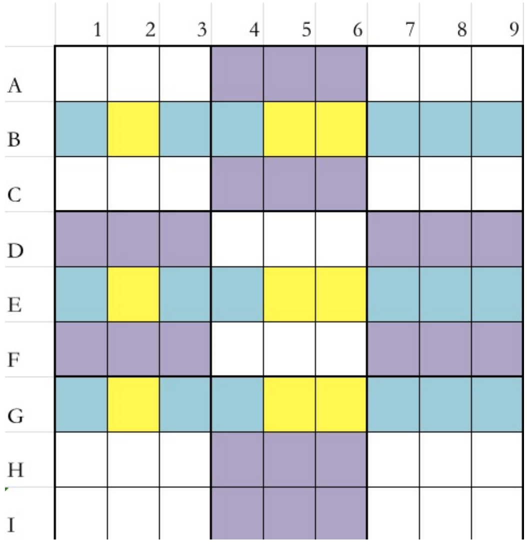



4.5.3

No matter which case appears above, we can infer that the red area of line B and line H can no longer be the number 9. The number "9" can be removed from the candidate number of red squares.

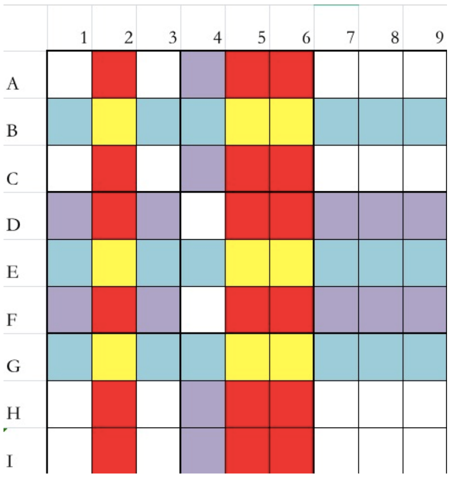
**Three-chain deletion method**

The triple-chain pruning method extends the rectangular vertex pruning method so that we can refer to the rectangular vertex pruning method for an easier understanding of the triple-chain pruning method. Use "find out the situation that a certain number only appears in the same three rows in a certain three columns, and then delete the number from the other candidates in these three rows"; Or the method of "finding out the situation that a certain number only appears in the same three columns in a specific three rows, and then deleting the number from other candidates in these three



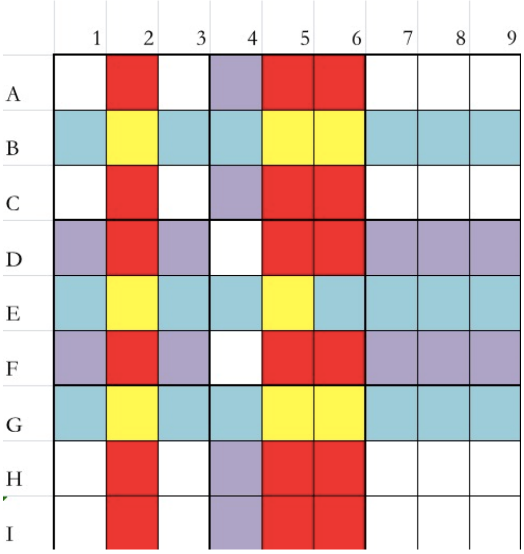

4.6.1

If the number "1" may appear in the yellow square of line B, line E and line G, it is in line with "the situation that a certain number only appears in the same three lines in a certain three columns" and meets the requirements of the three-chain column deletion method.




4.6.2

The red squares do not contain the candidate number "1".




4.6.3

This is a variation of the previous picture. The "1" in one row can only be placed at two positions in this row. Processing is the same as above, and the candidate number "1" can be excluded in red squares.

**Key number deletion method**

In the later stage of problem-solving, when no progress can be made by using the aforementioned unique candidate number method, unique implicit candidate number method, block deletion method, number pair deletion method, implicit number pair deletion method, triple chain number deletion method, implicit triple chain number deletion method, rectangular vertex deletion method and triple chain row deletion method, the critical number deletion method can be considered. The critical number deletion method is to find a number in the later stage, which only appears twice in the row (or column, nine squares). We assume that this number is in one of the lattice classes and continue to solve it. If there is an error, we will confirm that our hypothesis is wrong. If it is still challenging to solve the problem, let us assume that this number is in another square to see if we can get the error. This is the critical number deletion method. The essence of the critical number deletion method is a process that allows us to test one by one, gradually eliminate the impossible candidates, and then solve them.

**Other Applications of These Solutions**

However, apart from Sudoku, many excellent puzzles have numerous supporters and are inextricably linked with Sudoku. Sudoku lovers cannot miss these excellent logical reasoning games because their principles are similar to Sudoku or just derived from Sudoku. Here are some puzzles:

Diagonal Sudoku, Sudoku-X: The numbers on two big diagonals are not repeated based on the standard Sudoku rule. In this way, many manners can be applied from standard Sudoku, such as X-wings, implicit unique candidate number method and so on, since these two big diagonals also can be regarded as an extra row and column.

Jigsaw Sudoku: Compared with the standard Sudoku, the palace became irregular. Players should fill in nine or n numbers not repeated in the corresponding zigzag boxes and ensure that they are not repeated horizontally or vertically. As for this puzzle, maybe we can still apply those methods, but we need to do some alternations. For example, the name of X-wings in this game is another shape, maybe Jigsaw-wings.

Multi Sudoku: Each puzzle consists of two or more Sudoku grids overlapping, such as standard Sudoku puzzles or mixed Sudoku puzzles. These grids have one or more palaces overlapping. The purpose of the game is to solve each grid through its rules. Tips: Overlapping areas must meet the rules of their grid at the same time. In these puzzles, the methods are diverse, even we can combine different methods to solve this Sudoku, so it is challenging for people to manage the techniques.

Killer Sudoku, Sum Sudoku: Based on the standard Sudoku rule, the number in the upper left corner of each dashed box represents the sum of all the numbers in the dashed box, and the numbers in each dashed box are not duplicated. Diagonal Sudoku caused different areas, saw-tooth Sudoku broke the square formula, and killer Sudoku caused more computational

Sudoku. With more limitations, the methods in standard Sudoku can also be used in this puzzle.

Numbering: Like Sudoku, Numbering is a lively and straightforward game. You only need to connect companions who belong to the same number with a line. However, this game looks very simple, but it is profound.

Kenken: The purpose of the game is to fill spaces with numbers 1 to N(N is the number of rows and columns in the grid) so that the numbers in each row and column are not repeated, and the upper left corner of each thick wireframe represents the algorithm and calculation results of the numbers in the thick wireframe. In the thick frame of Sudoku, the same number may be used more than once.

All games above are fascinating, and they are efficient ways for us to exercise our logic and brain speed. [xii]

## Conclusion

Sudoku is an increasingly popular puzzle around the world. The rules of the puzzle are easy to be got, and not based on mathematics. There are various methods to solve the Sudoku concerning intuition techniques and candidate number techniques. The essay has briefly introduced several simple and reasonable methods containing primary exclusion method, block exclusion method, only remainder solution, rectangular exclusion method, unit exclusion method, remainder test method, unique candidate number method, unique implicit candidate number method, block deletion method, number pair deletion method, implicit number pair deletion method, triple chain number deletion method, implicit triple chain number deletion method, rectangular vertex deletion method, triple chain column deletion method and key number deletion method. What is more, some researches show that Sudoku is an efficient way to enhance our brain's ability, especially for their memory or logic. In that case, our children and elderly are appropriate to play this puzzle. Besides, the logic and methods in Sudoku are worth to be applied in other puzzles and games, such as Diagonal Sudoku, Jigsaw Sudoku, Multi Sudoku and Killer Sudoku.

In conclusion, Sudoku is a good choice for the elderly and children because it can strengthen the logic of their brain and exercise their ability of memory, especially for those high age people. Besides, the methods contained in this puzzle can be applied in many other puzzles or games, as these puzzles have resembled principles. However, it is hard for us to apply these methods in other puzzles unless we manage these skills deeply and flexibly.

**Conflict of Interests:** the author has claimed that no conflict of interests exists.

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